Dynamics

Unit 4

Contents

You have already come across many of the concepts covered in this unit. The concepts will be extended using what you learned about vectors in Unit 2. You covered kinematics in the last unit, which is describing motion. In this unit you will be considering dynamics, which looks at the causes of motion. Dynamics is applied in every branch of physics.

4.1 The force concept

By the end of this section you should be able to:

- Identify the four basic forces in nature.
- Define and describe the concepts and units related to force.

You should know from your earlier studies that a force is something that can change the speed and direction of movement of a body as well as changing the shape of a body. A force can act in any direction, which means that force is a vector.

There are four basic forces in nature:

- gravity: purely attractive force, which can act over long distances
- electromagnetism: attractive and repulsive force, which acts on charged particles
- weak nuclear force: acts on the scale of the atomic nucleus
- strong nuclear force: stronger than the electromagnetic force on the scale of the atomic nucleus – keeps protons and nuetrons bound together in the nucleus.

In Unit 2 we looked at adding forces together. Forces can act in all directions. Sometimes we want to resolve a force into components that are perpendicular to each other. These components usually are aligned to the components of the frame of reference we use. The components are usually horizontal and vertical vectors. We know from adding vectors together that the combination of these two forces has the same effect.

Figure 4.1 shows an example of where resolving the forces can be useful. The man is pulling a heavy load using a rope. The rope provides the force needed to move the load along the ground. The rope is at an angle to the horizontal, so the force it provides has a horizontal component and a vertical component.

It is only the horizontal component of the force that causes the load to move. (The vertical component tends to lift the load upwards.) This tells us that the man's force would have more effect if he pulled horizontally; the steeper the angle of the rope the less effective he will be, because the horizontal component will be smaller.

Discussion activity

Discuss what distances the four forces act over.

What effect do they have on our surroundings?

Draw up a table to summarise the results of your discussion.

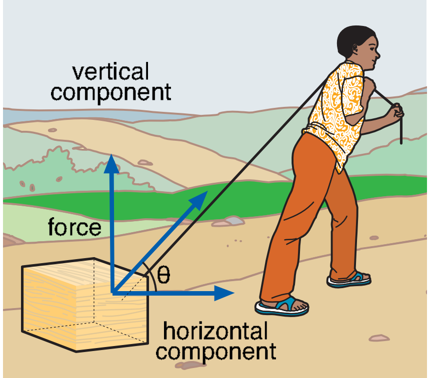

Figure 4.1 The force pulling the load is at an angle to the horizontal. It is the horizontal component of the force that causes the load to move along the ground.

Figure 4.2 Crossing this river is tricky when the river is flowing fast

You can work out the horizontal and vertical components of the force by resolving the force into its horizontal and vertical components. If the force used is *F*, then

vertical component, $F_y = F \sin \theta$

horizontal component, $F_x = F \cos \theta$

Look at Figure 4.2. The man is pulling a load across the fast-flowing river. The river pushes the raft downstream. To make the raft go straight across, the man has to pull at an angle. One component of his pulling force is needed to counteract the force of the moving water. The other component, at right angles to the riverbank, makes the raft move across to the other side.

Worked example 4.1

A man is pulling a string, which is attached to a box. The string is at an angle of 40° to the horizontal. The force applied to the box is 50 N.

What is the force needed to move the box?

First draw a diagram to show the force and its direction (Figure 4.3).

The component of the force that causes the box to move is the horizontal component.

Using trigonometry

horizontal component = 50 cos 40° = 38.3 N

It takes 38.3 N to move the box.

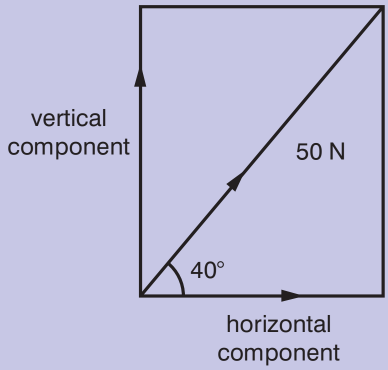

Figure 4.3

Summary

In this section you have learnt that:

- A force can change the speed and direction of a body.
- Force is a vector.
- A force can be resolved into horizontal and vertical components.

Review questions

1 Look at Figure 4.1. The man is using a force of 110 N to pull the box. He is pulling at an angle of 50° to the horizontal.

What is the force needed to move the box?

2 Look at Figure 4.2. The man is pulling on the rope with a force of 175 N. He is pulling at an angle of 40° to the flow of the river.

a) Draw a diagram to show the directions of the forces.

b) What is the force he is exerting to move the raft across the river?

Activity 4.1

- Attach a length of string to a box on the floor. Attach a newtonmeter to the end of the string.
- Hold the string at an angle to the ground and measure the angle.
- Pull the box until it moves – record the force needed to make the box move.
- Repeat for the string held at different angles.
- Work out the horizontal component of the force for each attempt to move the box. What do you notice?

4.2 Basic laws of dynamics

By the end of this section you should be able to:

- Define, and when appropriate give examples of, such concepts as gravity and Newton's law of universal gravitation.
- Describe how Newton's laws of motion and his law of universal gravitation explain the phenomenon of gravity and necessary conditions of 'weightlessness'.
- Define the term dynamics.
- Define and describe the concepts and units related to coefficients of friction.
- Use the laws of dynamics in solving problems.
- Interpret Newton's laws and apply these to moving objects.
- Explain the conditions associated with the movement of objects at constant velocity.
- Solve dynamics problems involving friction.
- State Newton's universal law of gravitation.
- Analyse, in qualitative and quantitative terms, the various forces acting on an object in a variety of situations, and describe the resulting motion of the object.

KEY WORDS

dynamics *the study of what causes objects to move*

Recap of Newton's laws of motion

In Grade 9, you learned about the basic laws of **dynamics**. Newton's three laws of motion are at the centre of this. They are:

- First law: a body will continue in its state of rest or uniform motion unless a force acts on it.
- Second law: when a force acts on a body, the body is accelerated by this force according to the relationship $F = ma$ where F is the force acting on the body, m is the mass of the body and a is the acceleration of the body.

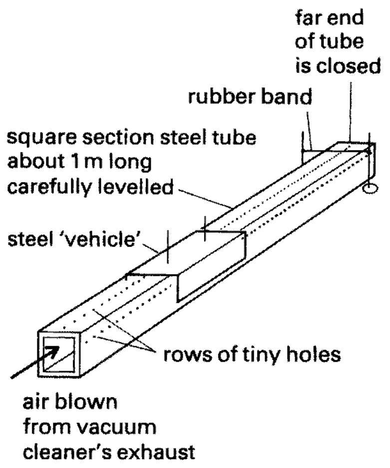

Figure 4.4 *An air track can be used to demonstrate Newton's laws of motion.*

Discussion activity

What other ways of demonstrating Newton's laws can you think of that use the trolleys shown in Figure 4.5?

Discussion activity

Consider the two students in Figure 4.5. Consider the forces exerted by each student as vectors.

- What do they add up to?
- What about the accelerations?

- Third law: when a body exerts a force on a second body, the second body exerts a force which is equal in size but in the opposite direction to the force exerted by the first body.

Newton's third law can also be expressed as: 'for every action there is an equal and opposite reaction'.

Demonstration of Newton's laws

We can demonstrate Newton's laws using an air track, as shown in Figure 4.4. The air track reduces friction by keeping the 'vehicle' clear of the tube from the air blowing out of the holes in the tube. The 'vehicle' is supported like a hovercraft.

If we push the vehicle and let go, it will keep going because there is no friction to slow it down. If the rubber band was not there to stop it, it would carry on at the same speed.

We can also demonstrate Newton's third law. Two students stand on wheeled trolleys, as shown in Figure 4.5. One student pushes on the other. What happens? Both students move, but in opposite directions. This shows that as the first student exerts a force on the second one, the second student is exerting a force on the first one which is equal in size and opposite in direction.

When the students start moving, their acceleration is determined by Newton's second law.

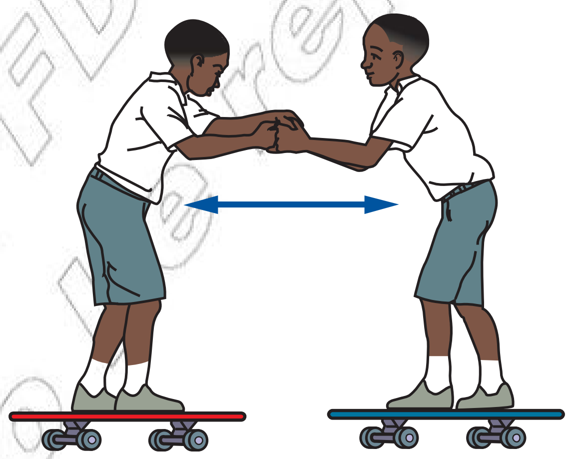

Figure 4.5 *What will happen when one student pushes on the other?*

Newton's second law

As force is a vector – it has direction – then the acceleration produced by it will also have a direction and will also be a vector. Mass does not have a direction – it is a scalar. The acceleration will be in the same direction as the force because mass is positive and cannot be negative.

The right-hand side of the equation $F = ma$ is a vector multiplied by a scalar, which you came across in Unit 2.

Worked example 4.2

Look at Figure 4.6. The mass of the lorry is 10 tonnes. What will happen to the lorry?

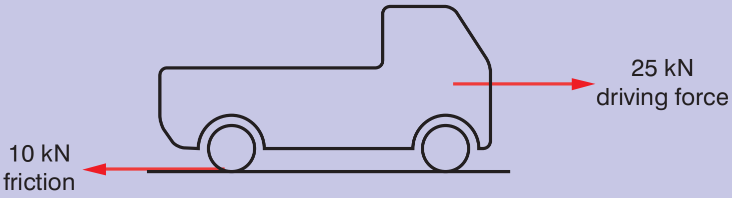

Figure 4.6

Consider the horizontal and vertical forces.

Assume that the positive horizontal direction is to the right, and the positive vertical direction is upwards.

There is a force downwards, which is the weight of the lorry, which is balanced by an upwards force from the road, so the net vertical force (F_y) is zero.

There are two horizontal forces acting on the lorry: a driving force acting to the right, +25 kN, and a friction force, acting to the left, −10 kN.

So the net force, F_x = 25 kN − 10 kN = 15 kN

There is a net force to the right (positive x-direction), which will produce an acceleration to the right.

So we need to use Newton's second law $F = ma$, but the net force only acts in the x-direction.

$a_x = F_x/m$ = 15 kN/10 000 kg = 1.5 m/s^2

(Remember that the mass of the truck needs to be converted to kilograms.)

The truck will accelerate to the right at 1.5 m/s^2.

Worked example 4.3

A ball of mass 0.5 kg is held on a slope of 20° to the horizontal. The ball is let go. What will the acceleration of the ball be?

First draw a diagram (Figure 4.7). We need to find the component of the weight of the ball that is acting down the slope – this will be the force that is accelerating the ball.

From the diagram we can see the component of the force that is parallel to the slope is $mg \sin 20°$

$F = ma$

so $ma = mg \sin 20°$

$a = g \sin 20°$

$= 9.8 \times 0.342 = 3.35$ m/s^2 at an angle of 20° below the horizontal.

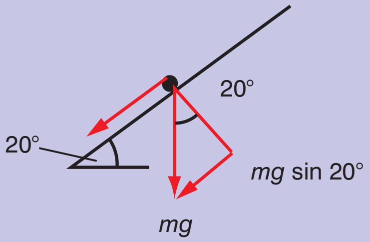

Figure 4.7

The ball is let go and it rolls down the slope.

Friction

Friction is the force that stops us from slipping when we walk. There are two types of friction: static friction and kinetic friction. **Static friction** is the friction between two surfaces when there is no movement. For example, when a car is not moving, the static friction between the tyres and road stops the car from sliding.

Kinetic friction is the friction between two surfaces when one of them is sliding over the other. For example, when you push a box along the floor, there is kinetic friction between the box and the floor when the box is moving.

Static friction

Imagine that you try to push a box along a table. With a small force, the box will not move. The force you apply is equal to the frictional force – if it was not, the box would move.

KEY WORDS

kinetic friction *the frictional force between two objects sliding over each other*

static friction *the frictional force between two objects that are trying to move against each other but are not yet moving*

KEY WORDS

limiting friction *the maximum value of static friction*

As you increase the force, you will reach a point where the box will begin to move – the frictional force reaches a maximum value. At this maximum value, the friction is said to be **limiting**. If the box does not move and the friction is limiting, the box is in limiting equilibrium.

When the frictional force is at the maximum, the box will either be moving or on the verge of moving.

The coefficient of static friction is a number between 0 and 1, which represents the friction between two surfaces. The maximum frictional force (in limiting equilibrium) is:

$$F_s = \mu_s F_N$$

where F_s is the frictional force, μ_s is the coefficient of static friction and F_N is the normal force between the two objects, as shown in Figure 4.8.

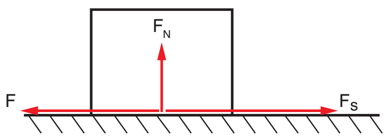

Figure 4.8 *Static friction*

Worked example 4.4

A box that weighs 60 N is on a ramp, which is inclined at 30° to the horizontal (Figure 4.9).

The box is in limiting equilibrium. Find the coefficient of friction between the box and the plane.

We need to resolve the weight of the box into components that are parallel and perpendicular to the plane, as shown in Figure 4.10.

The normal force is:

F_N = 60 cos 30°

In limiting equilibrium, the frictional force up the slope and the force down the slope are equal and opposite and we know that $F_s = \mu_s F_N$, so:

$\mu_s F_N$ = 60 sin 30°

Substituting for F_N:

μ_s60 cos 30° = 60 sin 30°

μ_s = sin 30°/cos 30°

= 0.5/0.866 = 0.58

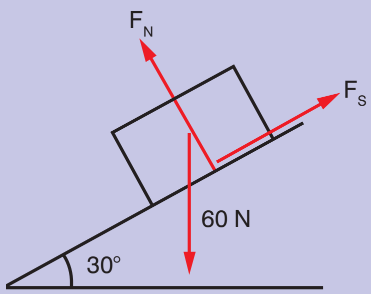

Figure 4.9

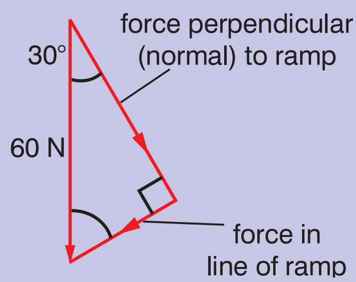

Figure 4.10 *Components of weight of box parallel and normal to the ramp*

Activity 4.2

You are going to find the coefficient of static friction between two surfaces – a box and a ramp.

- Measure the weight of the box.
- Put the box on the ramp. Tilt the ramp slowly until the box slides. Record the angle of the ramp.
- Repeat twice.
- Repeat for other materials.
- Work out the coefficient of friction between the surfaces.

Project work

Prepare a presentation on the importance of friction in everyday life. Where do we try to maximise friction and where do we try to minimise it?

Kinetic friction

The coefficient of kinetic friction is a number between 0 and 1, which represents the friction between two surfaces. The frictional force is:

$$F_k = \mu_k F_N$$

You should have found that static friction is greater than kinetic friction.

Activity 4.3

You are going to find the coefficient of kinetic friction between two surfaces – a box and a table.

- Measure the weight of the box, as you did in Activity 4.2.
- Put the box on the table. Pull the box along the table at a steady speed, as shown in Figure 4.11. Record the force needed to pull the box. (You may need to practise pulling the block at a steady speed a couple of times.)
- Repeat twice at the same speed.
- Repeat for different speeds.
- Repeat for other materials.
- Work out the coefficient of kinetic friction between the surfaces.

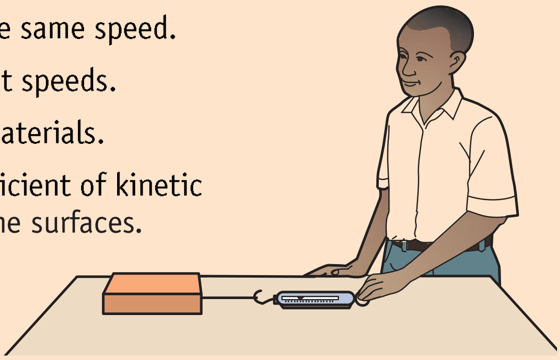

Figure 4.11 *Pulling a block along a tabletop*

Worked example 4.5

A box is pulled along a flat table. The force used to move the box along the table is 7 N.

The mass of the box is 2 kg.

Work out the coefficient of kinetic friction.

Draw a free body diagram to show the forces (Figure 4.10).

The normal force

$F_N = mg = 2 \times 9.8 = 19.6$ N

$F_k = \mu_k F_N$ so

$\mu_k = F_k/F_N = 7/19.6 = 0.36$

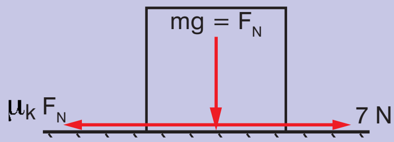

Figure 4.12

Discussion activity

In about 1600, Galileo carried out a 'thought' experiment on motion. The line between A and B represents a surface and the dot at A is a ball. What would happen in each of the three situations shown in Figure 4.13, with and without friction when the ball is released. Why do you think Galileo had to carry this out as a thought experiment?

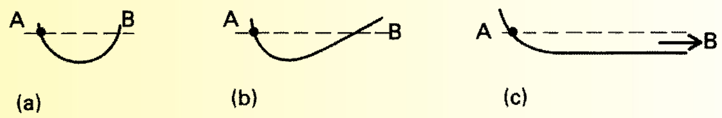

Figure 4.13 *Galileo's thought experiment on motion*

Activity 4.4

Repeat Activity 4.3, but this time with the block on a ramp. Measure the force needed to pull the block at a steady speed for different angles of the ramp.

What can you conclude from this?

Newton and gravity

In Unit 3, you learned more about Newton's law of universal gravitation.

Newton's law of universal gravitation states that if two masses M_1 and M_2 are a distance apart r, then the gravitational force between them is given by the equation:

$$F = \frac{GM_1M_2}{r^2}$$

where G is the universal gravitational constant.

We can use it and Newton's laws of motion to help explain what we call gravity. Consider a mass m held above the Earth's surface and then released (Figure 4.14). It will of course drop, gaining velocity all the time, as they are attracted to each other according to the law of universal gravitation.

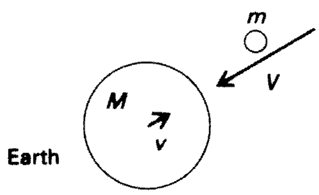

Figure 4.14 *Both masses gain momentum*

This the exact opposite of the case where two masses are moving apart from each other, but otherwise the principles are exactly the same:

$$Mv + mV = 0$$

The object accelerates rapidly towards the Earth, of course, but under the action of the same size force the opposite way, the Earth accelerates with its huge mass very slowly towards the object.

When an astronaut is in the International Space Station in orbit around the Earth, gravity is acting on both the International Space Station and the astronaut. The astronaut experiences the feeling of weightlessness because both the space station and the astronaut are falling continuously towards the centre of the Earth, but the sideways motion (the movement in orbit around the Earth) maintains the distance from the centre of the Earth.

In fact, eventually both the astronaut and the space station would fall to the Earth. In the space station, there is still a gravitational pull, but it is not as strong as it is on the surface of the Earth.

Summary

In this section you have learnt that:

- Newton's three laws of motion are fundamental to the movement of bodies.
- There are two types of friction: static friction and kinetic friction.
- The coefficient of static friction is found from the equation $F_s = \mu_s F_N$ where F_s is the frictional force, μ_s is the coefficient of static friction and F_N is the normal force.
- The coefficient of kinetic friction is found from the equation $F_k = \mu_k F_N$ where F_k is the frictional force, μ_k is the coefficient of kinetic friction and F_N is the normal force.

Review questions

1. A girl is on a bicycle cycling along a flat road. She applies a force to the pedals which produces a driving force of 1 kN. The mass of the girl is 65 kg and the mass of the bicycle is 20 kg.

   There is a combined force of 500 N from friction and air resistance.

   a) What will happen to the girl and bicycle.

   b) The girl now cycles up a slope of 10°. What will happen to the girl and bicycle now?

   (Hint: you need to consider the components of the forces that are acting in the plane she is travelling.)

2. Look at the diagram of the truck in Figure 4.15. The truck has a mass of 8 tonnes. What will happen to the truck?

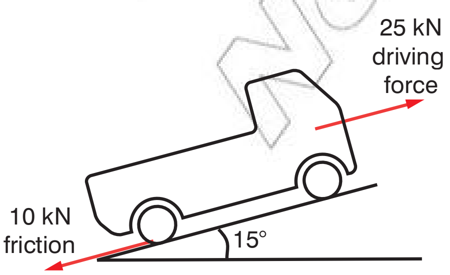

Figure 4.15

3. A book is on a ramp at angle of 10° to the horizontal. The book has a mass of 1.5 kg and the coefficient of static friction between the book and the ramp is 0.2.

   Work out the frictional force between the book and the ramp.

4. A man puts a brick down on a concrete slope. The brick just starts to slip.

   The angle of the slope is 35°. Work out the coefficient of static friction between the brick and the concrete.

5. A large table has a mass of 150 kg. The coefficient of static friction between the table legs and the ground is 0.45. The coefficient of kinetic friction between the table legs and the ground is 0.4.
   a) What force is needed to start the table moving?
   b) What force is needed to keep the table moving?

6. A person is dragging a 15 kg box along flat ground using a length of rope. The rope is at angle of 30° to the horizontal and the person is pulling with a force of 58 N.

   Work out the coefficient of kinetic friction between the box and the ground.

7. What conditions are associated with an object that is moving at a constant velocity?

8. A truck is facing down a slope of 5°. The truck has a mass of 3000 kg. The driver lets of the brake and the truck accelerates down the slope. What is the acceleration of the truck?

   What is the acceleration as a vector in component form?

9. A car is on a slope of 10° above the horizontal. A force of 1000 N is applied to the car up the line of the slope. The car has a mass of 500 kg.

   What is the acceleration of the car?

   Give your answer in vector form.

4.3 Law of conservation of linear momentum and its applications

By the end of this section you should be able to:

- Describe the terms momentum and impulse.
- State the law of conservation of linear momentum.
- Discover the relationship between impulse and momentum, according to Newton's second law.
- Apply quantitatively the law of conservation of linear momentum.

KEY WORDS

linear momentum *the product of the mass and velocity of a particle*

You learned in Grade 9 the meaning of the term momentum. The greater the mass and velocity of an object, the greater its momentum. Momentum is defined by the equation:

linear momentum = mass × velocity or $p = mv$

The term linear is used to distinguish it from angular momentum. A body has angular momentum when it is spinning. The units of momentum are kg m/s.

As velocity is a vector and mass is a scalar, momentum is also a vector because when you multiply a vector by a scalar, you get a vector. The linear momentum of a body has the same direction as the velocity of the body.

So far we have used a simplified form of Newton's second law of motion. The full version includes momentum and is:

- **If a resultant force acts on a body, it will cause that body's momentum to change. The momentum change occurs in the direction of the force, at a rate proportional to the magnitude of that force.**

We can also *express* momentum as a vector. According to Newton's second law, the direction of the momentum will always be in the same direction as the velocity because mass always has a positive value.

In vector form: $\mathbf{p} = m\mathbf{v}$

We can also state Newton's second law in terms of momentum. The net force can be expressed in terms of change in momentum divided by time, or the rate of change of momentum.

$\mathbf{F}_{net} = \Delta\mathbf{p}/\Delta t$

Activity 4.5

Consider a ball of mass 150 g thrown into the air with a velocity of 15 m/s at an angle of 60° to the horizontal.

- What happens to the momentum of the ball?
- What do you notice about the momentum when you express it in vector form?

Activity 4.6

Look at the units of each of the quantities in the two equations on the left. Check that the units on the left-hand side of the equation are the same as the ones on the right-hand side.

Law of conservation of linear momentum

When two masses push each other apart, you can use Newton's third law of motion to predict the movement of one mass when you know the velocity of the other mass.

We can demonstrate this with two similar free-running trolleys – you could use something like the ones shown in Figure 4.16.

Add extra mass to one of them so that it has double the mass of the other one. Compress a stiff spring between them, keeping it squashed with a loop of thread holding it together, as shown in Figure 4.16.

Burn through the thread and the trolleys will push each other apart. All the time this is happening the trolley with double the mass will be accelerating at half the acceleration of the other, so by the end it will only have half its velocity, but in the opposite direction.

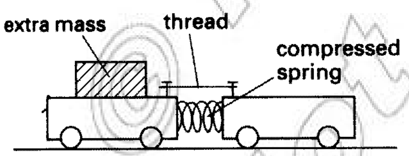

***Figure 4.16** Demonstrating the conservation of linear momentum*

Worked example 4.6

Consider a large mass and a small mass which are pushing one another apart, as shown in Figure 4.17. The small mass moves away at a velocity of 10 m/s to the right. What is the recoil velocity of the large mass?

According to Newton's third law, the forces exerted by each mass on the other are equal in magnitude but opposite in direction. Each force causes the momentum of the body on which it acts to change. During the time which they push each other apart, A and B gain momentum. Taking our frame of reference to be positive to the right, A gains momentum p_A and B gains momentum p_B.

Using Newton's third law: $p_A = -p_B$

but $p_A = m_A v_A$ and $p_B = m_B v_B$

so $m_A v_A = -m_B v_B$

$v_A = -m_B v_B / m_A$

Substitutuing the values into the equation

$v_A = -2 \text{ kg} \times 10 \text{ m/s} / 5 \text{ kg}$

$= -4 \text{ m/s}$

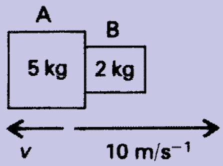

Figure 4.17

This example illustrates a particular case of a general principle – the conservation of linear momentum. The principle is:

- **If two bodies collide or push each other apart and no forces act except for each one pushing on the other, the total momentum of the two bodies does not change.**

Looking at the worked example, at the start the total momentum of both bodies was zero. After the bodies have moved apart the total momentum is

$$\boldsymbol{p} + -\boldsymbol{p} = 0$$

So the total momentum of the two bodies has not changed.

In Grade 9, we applied the law of conservation of momentum in one dimension. Now we will apply the law in two dimensions. The principle is:

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

where $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{v}_1$ and $\mathbf{v}_2$ are vectors.

Activity 4.7

Find two students with approximately the same mass. Ask them to stand on platforms with wheels, facing each other, as shown in Figure 4.18.

One student pushes the other away gently, in an attempt to make him or her move away. What happens?

Does it make any difference which student does the pushing, or if both push?

Try again with students of different masses.

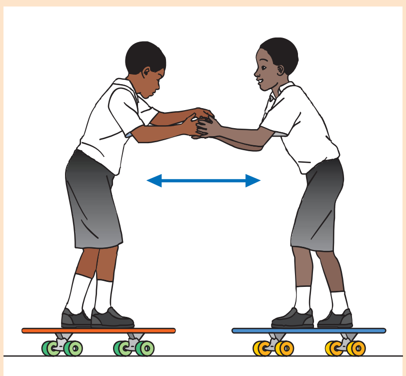

Figure 4.18 *What will happen if one student pushes on the other?*

Worked example 4.7

Two roads are perpendicular and meet at a junction.

Car A of mass 1000 kg travels along one road at 20 m/s due North.

Car B of mass 1300 kg, travels due West along the other road at 16 m/s.

At the junction the cars collide and move together.

a) Write down the initial velocities of the cars before collision in vector form.

b) Find the velocity of the cars after collision.

First draw a diagram of the cars before collision (Figure 4.17).

a) Assume North and West to be positive in our frame of reference. The initial velocities are:

car A: $\begin{bmatrix} 0 \\ 20 \end{bmatrix}$ m/s

car B: $\begin{bmatrix} 16 \\ 0 \end{bmatrix}$ m/s

b) momentum before collision $= 1000 \begin{bmatrix} 0 \\ 20 \end{bmatrix}$ kg m/s

$+ 1300 \begin{bmatrix} 16 \\ 0 \end{bmatrix}$ kg m/s

$= \begin{bmatrix} 20\ 800 \\ 20\ 000 \end{bmatrix}$ kg m/s

momentum after collision $= (1000 + 1300)\ \mathbf{v}$ kg m/s

$= 2300\mathbf{v}$ kg m/s

So $\mathbf{v} = \begin{bmatrix} 20\ 800/2300 \\ 20\ 000/2300 \end{bmatrix}$ m/s

$= \begin{bmatrix} 9.04 \\ 8.70 \end{bmatrix}$ m/s

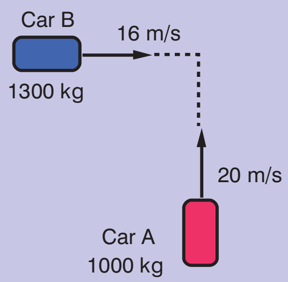

Figure 4.19

Activity 4.8

In each of the following cases, describe what happens and try to explain it in terms of the conservation of momentum.

- One student stands on a trolley. Another student who is standing on the floor throws a ball filled with sand to the student on the trolley.
- Two students are each on stationary trolleys. One of them has a ball filled with sand and throws it to the other student.

Impulse

In Grade 9 you learned that an impulse is when you change the momentum of an object. As the mass does not change, it is the velocity that changes when the momentum is changed. The equation is:

impulse = mass × change in velocity or

$I = m\Delta v$

You also learned that the impulse is related to the force used to change the momentum and the length of time the force was applied for. The equation is:

$I = F\Delta t$

The units of impulse are newton seconds or N s, which are the same as momentum.

Discussion activity

Put the two equations for impulse together. Can you see a link between them? What equation does it resemble?

Worked example 4.8

A ball of mass 500 g and velocity $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ m/s bounces off a vertical wall.

a) What is the impulse of the ball?

b) The ball changes direction in 0.1 s. What is the force exerted by the wall on the ball?

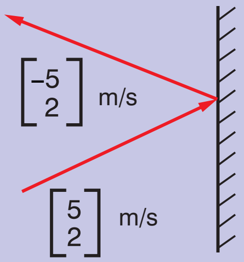

Figure 4.20

a) First draw a diagram showing the known variable (Figure 4.20).

Assume the frame of reference to be Cartesian coordinates, that is positive x-direction is to the right and positive y-direction is upwards.

Use the equation $p = mv$ to find the momentum before and after collision, and then $I = m\Delta v$ to find the impulse.

Momentum before collision = $0.5 \times \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1.0 \end{bmatrix}$ kg m/s

Momentum after collision = $0.5 \times \begin{bmatrix} -5 \\ 2 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 1.0 \end{bmatrix}$ kg m/s

Impulse = change in momentum = $\begin{bmatrix} 2.5 \\ 1.0 \end{bmatrix} - \begin{bmatrix} -2.5 \\ 1.0 \end{bmatrix}$ kg m/s

= $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ kg m/s

b) Use the equation $I = F\Delta t$ and rearrange it to find the force exerted by the wall.

$F = I/\Delta t$

Force exerted by wall = $\begin{bmatrix} 5 \\ 0 \end{bmatrix} /0.1 = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$ N

Newton's cradle

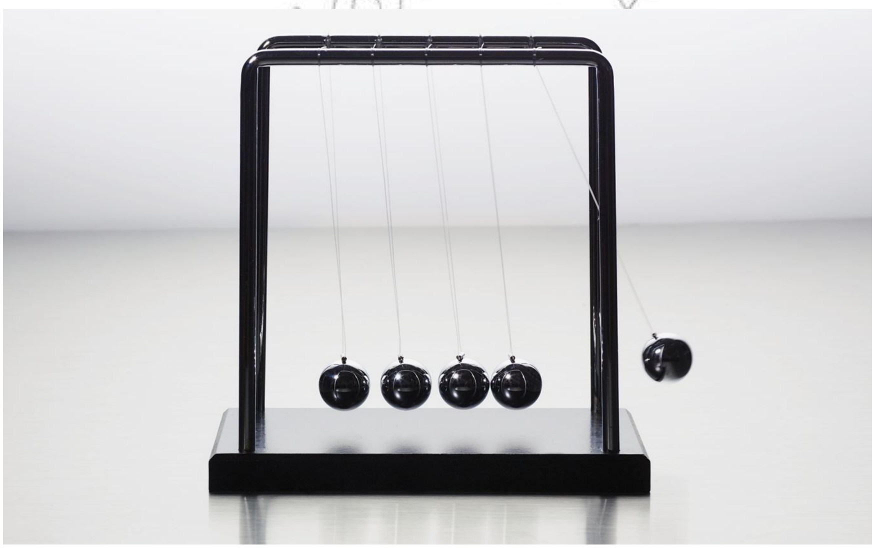

Figure 4.21 *A Newton's cradle relies on the law of conservation of linear momentum to work.*

DID YOU KNOW?

The recoil-less rifle

During the Second World War, a so-called recoil-less rifle was developed. It is so effective that a person can fire a shell that can pierce a tank's armour without suffering so much recoil that the person's shoulder is broken.

It does not defy Newton's third law, though. It fires two objects simultaneously in opposite directions. The shell comes out of the front of the gun, of course. At the back, a blast of compressed gases comes out, which is so intense that it could kill someone standing behind the rifle.

Seat belts

The effect of a force is to change a body's motion. Or, if you want to change the motion of a body, you need to apply a force.

When you drive a car and have to do an emergency stop, you put your foot hard on the brake pedal. The brakes exert a decelerating force on the car. You keep moving at the same velocity until you come across something that will provide the force to change your motion. If you are wearing a seat belt, this will exert a force on you which will change your motion. If you don't, you will hit the windscreen and the windscreen will provide the force to change your motion.

Crumple zones on cars

One way to reduce potential injuries in a car accident is to reduce the force exerted on the body to stop it. To reduce the force, you need to reduce the deceleration.

Another way of looking at this is to increase the time it takes for your momentum to change. One way of doing this is to have crumple zones on a car. As a car crashes, it compresses and so your momentum changes in a longer time, and the forces on you are less.

Activity 4.9

A car hits a wall at 20 m/s. The mass of one of the passengers is 75 kg.

The passenger comes to a halt in 0.01 s.

A second car hits a similar wall at the same speed. This car has crumple zones at the front. As the car hits the wall, the crumple zone works and the passenger comes to a halt in 0.1 s (Figure 4.22).

Draw free body diagrams to show the forces on the passengers.

Analyse the forces and accelerations on the two passengers, using the impulse-momentum equations given above.

What conclusions can you draw about the passengers in the two cars? What implications does this have for safety features in a car.

Figure 4.22 *Crash testing a car*

Summary

In this section you have learnt that:

- Linear momentum is mass multiplied by velocity and is a vector.
- Total momentum of a system of bodies stays the same unless a force acts on them to change the momentum.
- The law of conservation of momentum can be used to help explain gravity.
- Impulse is the change in momentum.
- Impulse and momentum have the same units.
- Increasing the length of time to change the momentum reduces the size of the force needed to change the momentum.

Review questions

1. A ball of mass 4 kg falls on to the floor with a velocity of $\begin{bmatrix} 4 \\ -9 \end{bmatrix}$ m/s.

   It bounces off the floor with a velocity of $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$ m/s.

   What is the impulse of the ball?

   The ball changes direction in 0.15 s. What is the force exerted by the floor on the ball?

2. A boy drops a stone of mass 200 g from a height of 2 m.

   a) What is the momentum of the stone just before it hits the floor?

   b) What is the impulse of the stone?

   c) The stone comes to a halt in 0.05 s. What is the force exerted on the stone?

3. A bullet of mass 0.01 kg is fired with a velocity of $\begin{bmatrix} 200 \\ 0 \end{bmatrix}$ m/s into a sack of sand of mass 9.99 kg which is swinging from a rope. At the moment the bullet hits, the sack has a velocity of $\begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$ m/s.

   Work out the velocity of the bullet and sack just after the bullet hits the sack.

4. Particle A is travelling at a velocity of $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ m/s. It collides with particle B which has a velocity of $\begin{bmatrix} 10 \\ 8 \end{bmatrix}$ m/s. The particles move together. The mass of particle A is 2 kg and the mass of particle B is 3 kg.

   Find the velocity of the combined particles after the collision.

KEY WORDS

elastic collision *a collision between two particles where kinetic energy is conserved*

inelastic collision *a collision between two particles where kinetic energy is not conserved*

Activity 4.10

We can also demonstrate collisions using an air track, such as the one shown in Figure 4.4. How would you demonstrate elastic and inelastic collisions?

DID YOU KNOW?

Collisions are used in atomic and nuclear physics to investigate the properties of particles. Particles have been discovered by looking at the tracks of particles after they have collided.

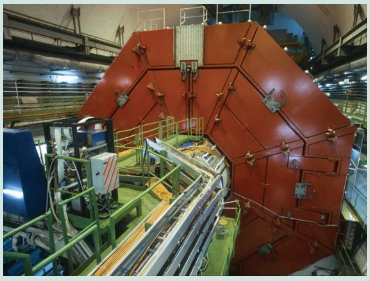

Figure 4.23 *Particles are collided in a particle accelerator like this one.*

4.4 Elastic and inelastic collisions in one and two dimensions

By the end of this section you should be able to:

- Distinguish between elastic and inelastic collisions.
- Describe head-on collisions.
- Describe glancing collisions.

In Grade 9, you learned that in a collision, momentum is conserved and you applied this in one dimension. Here, we will extend this to collisions in two dimensions.

Consider two identical masses approaching each other. One has velocity v, the other has velocity $-v$. Their combined momentum is $mv + (-mv)$, which is zero.

We know that after they collide their total momentum will still be zero, but we cannot predict exactly what will happen without further information.

If they were both balls of soft uncooked dough, they would merge together to form a single stationary lump of mass. As v is now zero, the total momentum is zero, and momentum has been conserved. This is an **inelastic collision**.

If they were two balls of hard spring steel, they would rebound off each other. The first ball would now have a velocity of $-v$ and the second one a velocity of $+v$. The total momentum is still zero, so momentum has been conserved. This is an elastic collision. Molecules in a gas collide like this.

In all collisions momentum is conserved. The difference between an elastic collision and an inelastic collision is that kinetic energy is conserved in an **elastic collision**, but not in an inelastic collision. The kinetic energy is transferred into other forms of energy.

Activity 4.11

Use two toy cars or laboratory trolleys. Attach magnets so that they repel each other when they collide.

- Push one car towards the other; observe how momentum is transferred to the second one.
- Try adding masses to the cars. How does this affect how they move after the collision?
- What happens if both cars are moving when they collide?
- Reverse one of the magnets so that the cars stick together when they collide. Does momentum still appear to be conserved?

Worked example 4.9

The cue ball in a pool game is travelling at $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ m/s. It collides with a pool ball which is stationary. After the collision, the pool ball moves with a velocity of $\begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$ m/s.

What is the velocity of the cue ball after the collision?

The cue ball has a mass of 90 g, and the pool ball has a mass of 100 g.

First draw a diagram showing the known variables and the unknown one (Figure 4.24).

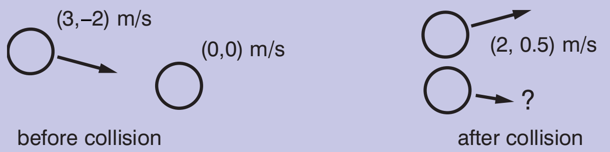

Figure 4.24

Momentum before collision = $0.09 \times \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 0.27 \\ -0.18 \end{bmatrix}$ kg m/s

Momentum after collision = $0.1 \times \begin{bmatrix} 2 \\ 0.5 \end{bmatrix} + 0.09 \times \begin{bmatrix} x \\ y \end{bmatrix}$ kg m/s

$= \begin{bmatrix} 0.2 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.09x \\ 0.09y \end{bmatrix}$ kg m/s

As momentum after collision = momentum before collision

$$\begin{bmatrix} 0.2 \\ 0.05 \end{bmatrix} + \begin{bmatrix} 0.09x \\ 0.09y \end{bmatrix} = \begin{bmatrix} 0.27 \\ -0.18 \end{bmatrix}$$

$$\begin{bmatrix} 0.09x \\ 0.09y \end{bmatrix} = \begin{bmatrix} 0.27 - 0.2 \\ -0.18 - 0.05 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.07/0.09 \\ -0.23/0.09 \end{bmatrix} = \begin{bmatrix} 0.78 \\ -2.56 \end{bmatrix} \text{ m/s}$$

So the velocity of the cue ball is $\begin{bmatrix} 0.78 \\ -2.56 \end{bmatrix}$ m/s

Head-on collisions

You learned about collisions in one dimension in Grade 9. A collision in one dimension is also known as a **head-on collision**. We can use the law of conservation of linear momentum and the fact that kinetic energy is conserved in an elastic collision.

Glancing collisions

A **glancing collision** is a collision in two dimensions. We apply the principles in exactly the same way as for a head-on collision, but in two dimensions.

Activity 4.12

Carry out some collisions with billiard balls on a smooth surface. Have one billiard ball stationary on the smooth surface. Roll another billiard ball down a ramp and onto the flat surface so that it collides with the stationary billiard ball.

- Work out the velocity of the first billiard ball before collision. How can you do this if you know the height of the ramp?
- Measure the velocities of the two balls after collision.
- Plan your experiment and carry it out.
- Can you use your results to predict the results of another collision?
- Write up a report of your experiment using the writing frame in Unit 1.

KEY WORDS

glancing collision *a collision in two dimensions, where the objects rebound in the same plane but not necessarily the same direction as the original motion*

head-on collision *a collision in one dimension, where the objects rebound on straight line paths that coincide with the original direction of motion*

Worked example 4.10

Car A collides with car B, which is stationary and then both cars move together.

Before the collision, car A had a velocity of 5 m/s. The masses of car A and car B are 1250 kg and 1500 kg, respectively.

What is the speed of the two cars after the collision?

First draw a diagram to show all the known variables and the unknown.

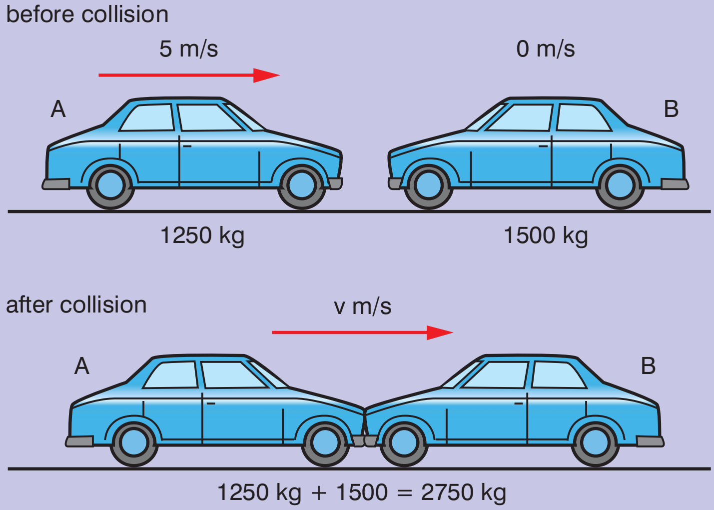

Figure 4.25

Use the law of conservation of momentum and the equation $p = mv$

$m_A v_A + m_B v_B = (m_A + m_B)v$

As $v_B = 0$, this term in the equation is zero. Rearrange the equation to find v_{AB}:

$v_{AB} = m_A v_A/(m_A + m_B)$

Substituting in the values:

$v_{AB} = (1250 \text{ kg} \times 5 \text{ m/s})/(1250 \text{ kg} + 1500 \text{ kg})$

$= 6250 \text{ kg m/s} \div 2750 \text{ kg} = 2.3 \text{ m/s}$

Summary

In this section you have learnt that:

- Kinetic energy is conserved in an elastic collision but not in an inelastic collision.
- The total momentum of a system of bodies before a collision is the same as the total momentum of the system after the collision.
- Problems involving collisions can be solved by considering the momentum before and after the collision.

Review questions

1. A is a sphere that is travelling with a velocity of (3, 7) m/s and had a mass of 5 kg.

   It collides with sphere B and both particles move together with a velocity of (1, 4) m/s after the collision. Sphere B has a mass of 4 kg.

   Find the velocity of B before the collision.

2. Two particles collide and come to rest. The first particle has mass 5 kg and a velocity (8, −9) m/s before the collision. The second particle has a mass of 2 kg.

   Find the velocity of the second particle before the collision.

3. A pool ball of mass 100 g and velocity (5, −4) m/s collides with a stationary pool bar of the same mass. After the collision, one of the pool balls has a velocity (2, −3) m/s.

   Find the velocity of the other pool ball.

4.5 Centre of mass

By the end of this section you should be able to:

- Define and describe the concepts and units related to torque.
- Describe centre of mass of a body.
- Determine the position of centre of mass of a body.

KEY WORDS

centre of mass *the point in a body from which the force of gravity on that body appears to be acting*

torque *the turning effect of a force round a point*

When you apply two forces to an object, as shown in Figure 4.26, the forces cause the body to rotate. This turning effect of the forces is called a moment or torque. Think back to the work you did on levers – you found that you could move a large mass using a lever if the distance you applied the load was a long way from the pivot. The forces are not acting in the same line. There are moments about the pivot.

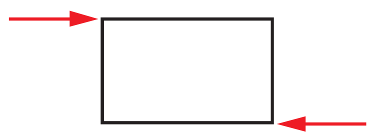

Figure 4.26 *The turning effect of two forces acting on a block*

The moment of a force or **torque** is defined as the magnitude of the force multiplied by the perpendicular distance from the point to the force of the line of action of the force. A moment, or the torque, is also the vector product of the force and the distance from the point. Torque has direction and so is also a vector.

Look at Figure 4.27 overleaf, which shows two masses connected by a rod. There is a point between the two masses m_1 and m_2 where they will balance, like a see-saw. The torque tries to move the objects around the point.

Activity 4.13

Set up a see-saw using a ruler and pivot. Put one mass on one side of the pivot. Can you use the principle of moments to predict where you will need to put double the mass on the other side of the pivot?

Activity 4.14

Stand a box on its smallest side on a flat surface. Push it over slightly and let it go.

Push it over a bit further and let it go. What happens?

Keep increasing the angle you push the box over to.

Can you find the angle where the box balances?

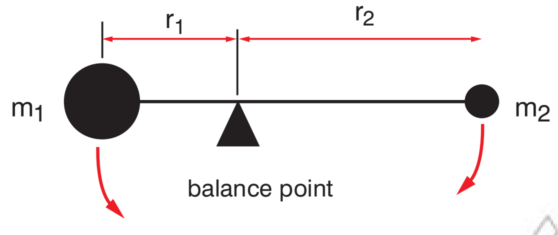

Figure 4.27 *Torque acting on masses*

In Figure 4.27, when the **centre of mass** is above the balance point, the rods balance. At this point the torque on m_1 is balanced by the torque on m_2. The torque on m_1 is trying to turn the rod anti-clockwise and the torque on m_2 is trying to turn the rod clockwise. Both force and displacement are vectors.

Worked example 4.11

Look at Figure 4.28. The two masses are balanced. What is the distance of the 2.5 kg mass from the balance point?

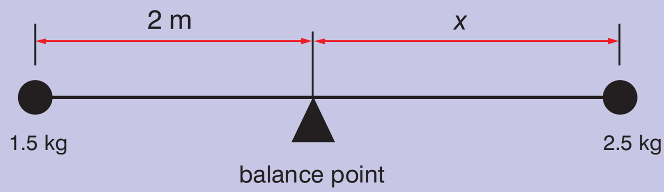

Figure 4.28 *Balanced system of two masses*

When the system is balanced, the moments about the balance point sum to zero.

On the left: $\tau_1 = -2 \text{ m} \times (1.5 \times 9.8) \text{ N}$

On the right: $\tau_2 = x \times (2.5 \times 9.8) \text{ N}$

$\tau_1 + \tau_2 = 0$

$-2 \text{ m} \times (1.5 \times 9.8) \text{ N} + x \times (2.5 \times 9.8) \text{ N} = 0$

$x = (2 \times 1.5)/2.5 = 1.2 \text{ m}$

The anticlockwise moment is:

$$\tau_1 = r_1 \times m_1 g$$

The clockwise moment is:

$$\tau_2 = r_2 \times m_2 g$$

When the system is balanced:

$$\tau_1 + \tau_2 = 0$$

We can also say that the system is balanced because the centre of mass is above the pivot point. Taking moments about a point helps us to explain the centre of mass. The position of the centre of mass explains why some objects are stable and others are not.

Finding the centre of mass

We can use moments to find the position of the centre of mass in a system.

Worked example 4.12

The diagram shows a rod with two masses attached to it. The distances from P are shown on the diagram. The rod is massless.

The moments of the two masses can be balanced by a single force F acting at a distance x from the left hand end of the rod. Calculate x.

Use the equation $\tau = rF$

Draw a diagram to show the forces acting on the rod.

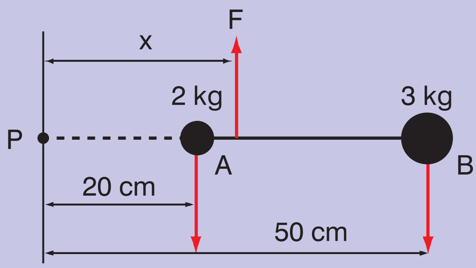

Figure 4.29

Anticlockwise moment about P = xF

Clockwise moment about P = $x_A m_A g + x_B m_B g$

When the rod is balanced, $F = m_A g + m_B g$

and $xF - x_A m_A g - x_B m_B g = 0$

So $x = \dfrac{(x_A m_A g + x_B m_B g)}{F}$

Substituting for F:

$x = (x_A m_A g + x_B m_B g)/(m_A g + m_B g)$

$= (x_A m_A + x_B m_B)/(m_A + m_B)$

Substituting in the values:

$x = (0.2\text{ m} \times 2\text{ kg} + 0.5\text{ m} \times 3\text{ kg})/(2\text{ kg} + 3\text{ kg})$

$= (0.4 + 1.5)/5 = 1.9/5 = 0.38\text{ m}$

So the force would need to be applied 38 cm from the left hand end of the rod.

The worked example above shows us that the centre of mass of the two masses is 38 cm from the left hand end of the rod. We can extend this result to find the centre of mass of any system of particles in two dimensions:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots}$$

where the distances are the distances from a common reference point.

When the centre of mass is directly above a point that is inside the base of an object, it will be stable, as shown in Figure 4.30. When the centre of mass is directly above a point which is outside the base of an object, the object will become unstable and fall over. The object will fall into a position where the centre of mass is directly over a point that is inside the new base of the object.

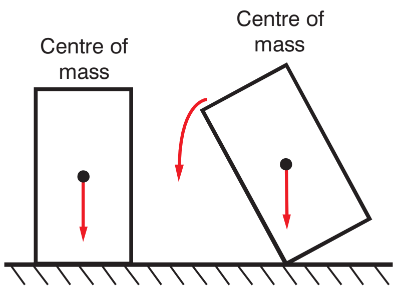

Figure 4.30 *How the position of the centre of mass affects the stability of an object*

Activity 4.15

You are going to find the centre of mass of a piece of card (Figure 4.31).

- Hang the card from one corner. Draw a vertical line from the point where the card is hanging.
- Repeat for different parts of the card.
- The place where the lines cross is the centre of mass.
- Cut an irregular shape from a piece of card and find its centre of mass.

Figure 4.31 *Finding the centre of mass of a planar object*

Activity 4.16

Kneel on a flat surface with your forearms flat on the surface and your elbows touching your knees (Figure 4.32a).

Place a short object such as a cigarette lighter or a matchbox at the tips of your outstretched fingers.

Now place your hands behind your back and try to tip the object over with your nose without losing your balance (Figure 4.32b).

Can you do it? Can you explain why?

Figure 4.32a

Figure 4.32b

DID YOU KNOW?

High jumpers use the Fosbury flop to help them get over the bar. When they do this, they bend their bodies so that their bodies clear the bar, but their centre of mass does not!

Figure 4.33 *A high jumper doing the Fosbury flop*

Summary

In this section you have learnt that:

- Torque is the turning effect of a force.
- The centre of mass is where the whole of the force of gravity on that body appears to be acting from.
- The centre of mass can be outside an object.
- Objects are stable when the centre of a mass is above a point that is inside the base of the object.

Review questions

1. A 250 g mass is 15 cm from the centre of a bar. Where should a 400 g mass be placed to balance the bar?
2. A bar is 20 cm long. A 100 g mass is placed on one end and a 150 g mass on the other. Where is the balance point of the bar?
3. Two masses (3 kg and 5 kg) are placed at opposite ends of a massless rod of length 1.5 m.

   Find the distance of the centre of mass from the 3 kg mass.
4. Find the distance of the centre of mass of the system shown in the diagram from point A. The rod is massless.

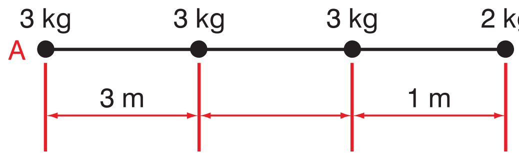

Figure 4.34

5. Find the centre of mass of each of the following systems with respect to corner O. The frame joining the masses is massless.

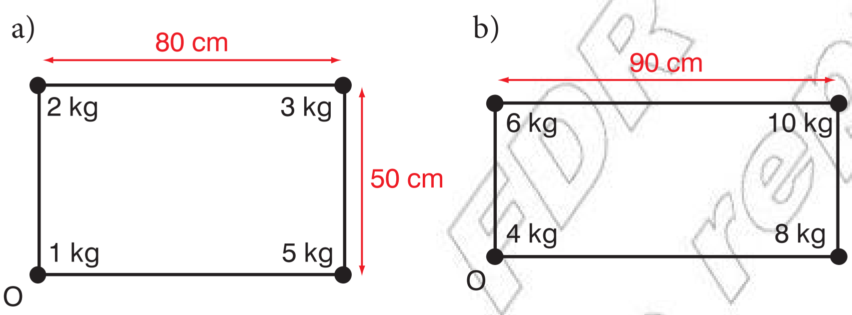

Figure 4.35

6. Calculate the mass of mass A in the diagram. The frame joining the masses is massless.

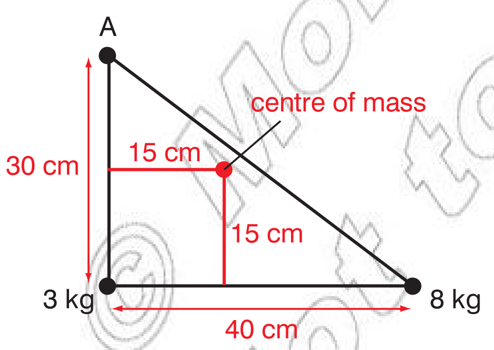

Figure 4.36

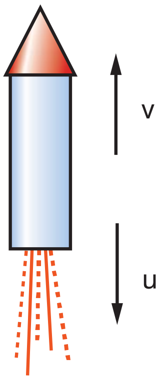

Figure 4.37 *A rocket*

Activity 4.17

Consider a rocket as shown in Figure 4.37. In small groups discuss the following:

- What happens to its mass?
- What happens to the acceleration of the rocket?
- How does the acceleration change while the engine is running?
- Do you think the rate of change of momentum is constant?

4.6 Momentum conservation in a variable mass system

By the end of this section you should be able to:

- Describe explosions and rocket propulsion in relation to momentum conservation.

A rocket uses Newton's third law of motion to move. The upwards force on the rocket is balanced by the downwards force from the gases going out through the rocket nozzle (Figure 4.37). However, we cannot apply Newton's second law of motion directly because the mass of the rocket is not constant. The mass of the rocket decreases as more gases are pushed out of the nozzle of the rocket. This also means that the rocket is losing some momentum as the gases leave the rocket.

We can express Newton's second law in this form:

$$F + u\Delta m = ma$$

where F is the force on the rocket, u is the velocity of the exhaust gases relative to the centre of mass of the rocket, $\frac{\Delta t}{\Delta m}$ is the mass of exhaust gases expelled at a point in time (or the rate of change of mass of the rocket), m is the mass of the rocket and a is the acceleration (or the rate of change in v, velocity of the rocket).

When the rate of change of mass is zero, this becomes Newton's second law of motion.

We need a generalisation of Newton's second law, which we covered in Section 4.3:

force = rate of change of momentum

Worked example 4.13

A toy rocket produces a force of 10 N. The mass of the rocket at launch is 200 g.

What is the acceleration of the rocket?

First draw a diagram showing the forces on the rocket (Figure 4.38).

There is a force downwards on the rocket which is the weight of the rocket. So the net force on the rocket is:

$F = 10\text{ N} - 0.2\text{ kg} \times 9.8\text{ m/s}^2 = 10 - 1.96 = 8.04\text{ N}$

acceleration $= \frac{8.04}{0.5}\text{ m/s}^2 = 16.1\text{ m/s}^2$

So the acceleration of the rocket is 16.1 m/s² upwards.

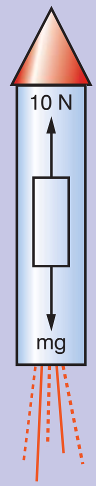

Figure 4.38

A raindrop is another example of a variable mass system. Droplets of condensed water vapour combine until they start falling through the cloud as a raindrop. The raindrops then get bigger as they combine with other droplets of water to form bigger raindrops. As the mass of raindrop increases, the momentum of the raindrop increases.

The force acting on the raindrop at a certain time can be expressed as:

$$F = \text{rate of change of momentum} = ma + \frac{u\Delta m}{\Delta t}$$

where F is the force on the raindrop, m is the mass of the raindrop, u is the velocity of the extra mass that has been added to the raindrop and u is the velocity of the extra mass.

We can also consider what happens in explosions.

Worked example 4.14

An 1 kg object explodes and breaks up into three pieces.

One piece has a mass of 400 g and has a velocity of $\begin{bmatrix} 15 \\ 10 \end{bmatrix}$ m/s.

A second piece has a mass of 250 g and has a velocity of $\begin{bmatrix} 25 \\ -10 \end{bmatrix}$ m/s.

What is the velocity of the third piece?

First draw a diagram to show the explosion (Figure 4.39).

We know that the total mass is 1 kg and the masses of the other two pieces are 400 g and 250 g, so the mass of the third piece is 350 g.

As the momentum before the explosion is zero, the total momentum after the explosion is also zero. So:

$$0.4 \times \begin{bmatrix} 15 \\ 10 \end{bmatrix} + 0.25 \times \begin{bmatrix} 25 \\ -10 \end{bmatrix} + 0.35 \times \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 \\ 4 \end{bmatrix} + \begin{bmatrix} 6.25 \\ -2.5 \end{bmatrix} + \begin{bmatrix} 0.35x \\ 0.35y \end{bmatrix} = 0$$

$$\begin{bmatrix} 0.35x \\ 0.35y \end{bmatrix} = \begin{bmatrix} -12.25 \\ 1.5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -35 \\ 4.3 \end{bmatrix} \text{ m/s}$$

So the velocity of the third piece is $\begin{bmatrix} -35 \\ 4.3 \end{bmatrix}$ m/s.

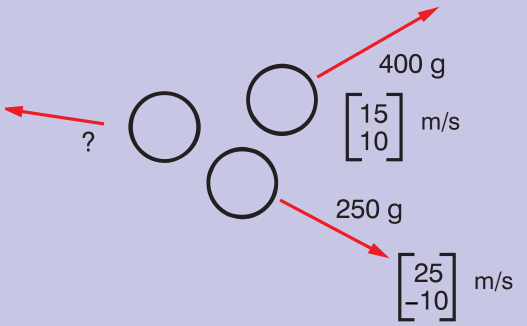

Figure 4.39

Activity 4.18

Attach a balloon to a trolley by holding it inside netting (Figure 4.40). Blow up the balloon and let the trolley go.

- What acceleration does the balloon give the trolley?
- What happens if you put a board immediately behind the trolley? Does this increase or decrease the acceleration of the trolley?

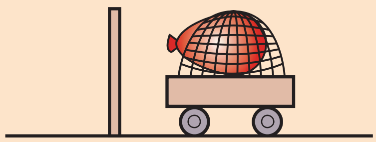

Figure 4.40

Summary

In this section you have learnt that:

- In a rocket, the mass changes while the engine is running.
- When the force is constant, the acceleration on a rocket increases while the engine is running.
- The law of conservation of momentum can be used to solve problems involving explosions.

Review questions

1. A toy rocket has a mass of 350 g at launch. The force it produces is 15 N and it is fired at an angle of 65° to the horizontal.
   What is the initial acceleration of the rocket as a vector?
2. A body of mass 50 kg explodes and splits into three pieces.
   The first piece has a mass of 10 kg and a velocity of $\begin{bmatrix} -18 \\ 23 \end{bmatrix}$ m/s, the second piece has a mass of 18 kg and a velocity of $\begin{bmatrix} 25 \\ -12 \end{bmatrix}$ m/s. What is the velocity of the third piece?
3. A body explodes and splits into three pieces. The first piece has a mass of 1.25 kg and a velocity of $\begin{bmatrix} 30 \\ -10 \end{bmatrix}$ m/s, the second piece has a mass of 3.25 kg and velocity of $\begin{bmatrix} -17 \\ 10 \end{bmatrix}$ m/s. The third piece has a velocity of $\begin{bmatrix} 35.5 \\ 40 \end{bmatrix}$ m/s
   a) What is the mass of the third piece?
   b) What was the total mass of the body before the explosion?

4.7 Dynamics of uniform circular motion

By the end of this section you should be able to:

- Interpret Newton's laws and apply these to moving objects undergoing uniform circular motion.
- Solve dynamics problems involving friction.

In Unit 3, we looked at uniform circular motion in both horizontal and vertical circles. Here we are going to look at the forces involved in keeping a body moving in uniform circular motion.

When a car is going round a bend, there is a force towards the centre of a circle that keeps the car moving in an arc of a circle round the bend. This force is provided by friction between the tyres of the car and the road.

One way of reducing the force needed to keep a car moving in a circle is to bank the track. Racetracks use this principle and well-constructed roads also use banking to make them safer.

Activity 4.19

You are designing a fairground ride where people will be spun round in circles on the inside of a drum. The floor of the drum is taken away leaving the riders stuck to the inner surface of the drum.

The drum then lifts up and rotates at an angle of 45° to the horizontal.

What data would you need to design the ride so that people are safe when the floor is removed and it tips up to 45° to the horizontal?

Worked example 4.15

A boy is sitting on the horizontal part of a roundabout 1 m from the centre. When the angular velocity of the roundabout exceeds 1 rad/s, the boy starts to slip.

Find the coefficient of friction between the boy and the surface.

First draw a diagram (Figure 4.41).

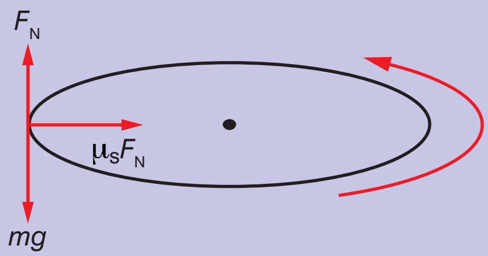

Figure 4.41

For the boy to just not slip or not to be moving with respect to the surface of the roundabout, the friction must be limiting. The forces in both the horizontal and the vertical directions are equal, as shown in the diagram

Normal force = mg

In the horizontal direction the frictional force is:

$F_s = \mu_s F_N = \mu_s mg$

but the force towards the centre is also his mass multiplied by his acceleration

$F = m\omega^2 r$

So $\mu_s mg = m\omega^2 r$ and

$\mu_s = \omega^2 r/g = 1^2 \times 1/9.8 = 0.11$

The coefficient of friction between the boy and the surface of the roundabout is 0.11.

Figure 4.42 *This cycle track makes use of banked curves, which means that the cyclists can go around the bends at each end of the track quicker.*

Worked example 4.16

A racetrack has a bend that is banked at an angle of 10° and has a radius of 200 m. A racing car has a mass of 750 kg and the coefficient of static friction between the tyres and the road surface is 0.6.

What is the fastest speed the racing car can go round this bend?

Draw a free body diagram showing the forces (Figure 4.43).

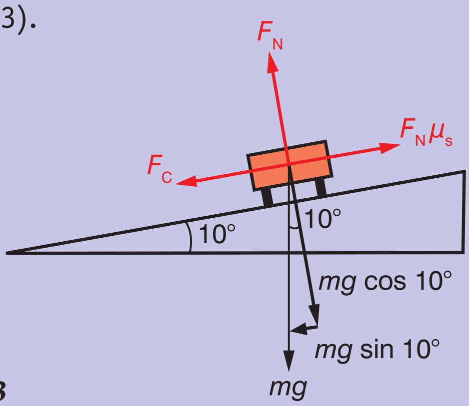

Figure 4.43

There is a centripetal force acting towards the centre of curvature of the bend, which is given by the equation

$F_c = \frac{mv^2}{r}$

The force normal to the plane of the racetrack is given by the component of the racing car's weight, which acts in this direction and is given by

$F_N = mg \cos 10°$

The frictional force between the racing car's wheels and the track is given by the equation:

$F_s = \mu_s F_N = \mu_s mg \cos 10°$

When the car starts to slip, the friction is limiting. The force of friction will be equal to the centripetal force acting on the car

$F_c = F_s$

So $\frac{mv^2}{r} = \mu_s mg \cos 10°$

$v^2 = \frac{\mu_s mg \cos 10° \, r}{m}$

$= \mu_s g \cos 10° \, r$

$= 0.6 \times 9.8 \text{ m/s} \times 0.985 \times 200 \text{ m}$

$= 1158 \text{ m}^2/\text{s}^2$

So $v = \sqrt{1158} = 34$ m/s

Activity 4.20

Set up a newtonmeter, string and mass up as shown in Figure 4.44. With one hand, hold on to the newtonmeter. With the other hand, hold on to the tubing and swing the mass in a horizontal circle.

- What force are you measuring?
- What do you notice about the force?
- Now try swinging the mass in a vertical circle. What do you notice about the force?

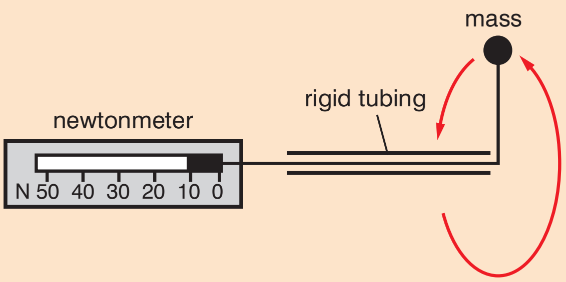

Figure 4.44

The forces acting on a body that is moving in a vertical circle are not constant. The centripetal force depends on the angle θ. The forces are shown in Figure 4.45.

The radial force (towards the centre of the circle) is $T - mg\cos\theta$.

The tangential force is $mg\sin\theta$.

We also know that the radial force is mv^2/r or $m\omega^2 r$

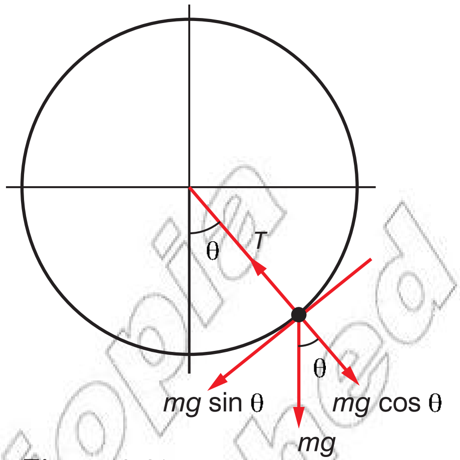

Figure 4.41

Worked example 4.17

A roller coaster moves through a loop. At an angle of 60° to the bottom of the loop it has a velocity of 14 m/s.

The mass of the roller coaster and riders is 750 kg. The diameter of the loop is 20 m.

What is the force acting on the roller coaster at an angle of 60° to the bottom of the loop?

Draw a free body diagram for when the roller coaster is at an angle of 60° (Figure 4.46).

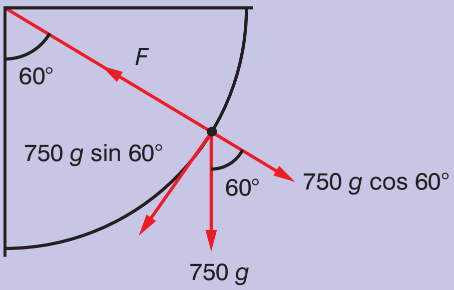

Figure 4.46

At this angle $F_N - mg\cos 60° = mv^2/r$

So $F_N = \frac{mv^2}{r} + mg\cos 60°$

Substituting in values for m etc:

$F_N = (750 \times 14^2/10) + (750 \times 9.8 \times 0.5)$

$= 14\ 700 + 3675$

$= 18\ 375$ N

Activity 4.21

Look at Figure 4.45. Consider the force T. How does it vary with the angle θ?

Consider how the equations above vary with the angle θ.

Activity 4.22

Design your own simple fairground ride, giving details of the forces involved and any minimum speeds required.

What is a reasonable maximum force for riders to experience?

Project work

Prepare a presentation on the applications of dynamics to an activity. Choose an activity such as sports, any ball game, seat belts, rocket travel and apply what you have learnt in this unit to your chosen activity.

Summary

In this section you have learnt that:

- The maximum velocity of a vehicle going round a bend can be increased by banking the surface.
- The radial force on a body is constant when the body is in uniform horizontal motion.
- The radial force on a body varies when the body moves uniformly in a vertical circle.
- The dynamics of uniform circular motion and friction can be used to solve many problems.

Review questions

1. A fairground ride consists of a large vertical drum that spins so fast that everyone inside it stays pinned against the wall when the floor drops away. The diameter of the drum is 10 m. Assume that the coefficient of static friction between the drum and the rider's clothes is 0.15.

   a) What is the minimum speed required for the riders so that they stay pinned against the inside of the drum when the floor drops away?

   b) What is the angular velocity of the drum at this speed?

2. A person is trying to ride a bike all the way round the inside of a pipe for a stunt in a film. The filmmaker wants to know what speeds are involved. The pipe has a diameter of 8 m.

   The mass of the bike and rider is 400 kg. The rider goes at a constant speed of 5 m/s.

   a) What is its acceleration at the bottom?

   b) What is the force on the bike at an angle of 30° up from the bottom?

   c) What is the minimum velocity at the top for the bike and rider to stay moving in a circle?

   d) Do the bike and rider have sufficient velocity to stay moving on a circle at the top?

End of unit questions

1. Construct a glossary of all the key terms in this unit. You could add it to the one you made for Units 1–3.
2. How does the law of conservation of momentum relate to Newton's second law of motion?
3. What is the difference between static friction and kinetic friction?
4. A jet engine generates 160 kN of force as it propels a 20 000 kg plane down a runway. If 40 kN of friction opposes the motion of the plane, how much time will it take for the plane to reach a speed of 33 m/s from rest.
5. A packing crate of weight 50 N is placed on a plane inclined at 35° from the horizontal. If the coefficient of static friction between the crate and the plane is 0.65, will the crate slide down the plane?
6. How does friction affect the maximum velocity of a car going round a bend?
7. Four billiard balls, each of a Mass 0.5 kg, are all travelling in the same direction on a billiard table, with speeds 2 m/s, 4 m/s, 8 m/s and 10 m/s. What is the linear momentum of this system?
8. A 60 kg man standing on a stationary 40 kg boat throws a 0.2 kg ball with a velocity of m/s. Assuming there is no friction between the man and the boat, what is the speed of the boat after the man throws the ball?
9. A spaceship moving at 1000 m/s releases a satellite of mass 1000 kg at a speed of 10 000m/s. What is the mass of the spaceship if it slows down to a velocity of 910 m/s?
10. The drawing in Figure 4.43 represents the steering wheel of a car. The driver exerts two equal and opposite forces on it as shown.

    Will the wheel be in equilibrium? Explain your answer.

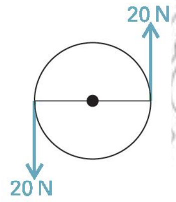

Figure 4.43

11. A roller coaster goes through a vertical loop. When are the forces on the riders:
    a) the smallest?
    b) the greatest?